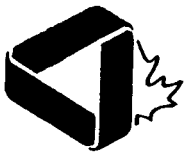

Canadian Mathematical Society

Société Mathématique du Canada

NOTES

Volume 18, No. 6

September/Septembre 1986

ISSN 0045-5164

Symbolic Mathematical Systems

and their Effects on the Curriculum

Bernard R. Hodgson (Laval) and Eric R. Muller (Brock)

The title of this short note may be familiar; it was the name of a special ICME V Session held in Adelaide. The papers and summaries of this session have been published in a special issue of SIGSAM Bulletin, one of the quarterly publications of the ACM (see [SIGSAM 85] in the following bibliography). Another set of thought-provoking articles appear in "The Influence of Computers and Informatics on Mathematics and its Teaching" (see [STRASBOURG 85]). It contains all the supporting papers for the ICMI Symposium held last year in Strasbourg.

What do these Symbolic Mathematical Systems offer?

1. They provide a rich environment for exploration where the student can experiment with different levels of generalization.

2. For some students they provide enrichment and motivation. (This may be especially true in the service course area.)

3. For students who require more examples and drills to assimilate concepts -- computer systems are well suited to provide numerous examples.

4. The student's exposure to more difficult examples helps to develop a sense of reasonableness, estimation, etc.

Introducing Symbolic Mathematical Systems in the early undergraduate courses requires a lot of work.

a) Computer considerations

   1. Most first time users find the systems difficult.

   2. A relatively large computer facility is required for high enrolment first and second year courses.

b) Mathematics considerations

   1. Problems in tests and course work are selected using many criterion, one of which is reasonable completion time. With symbolic systems reasonable time is extended. It is easy to claim that one can do more realistic problems -- it is very time consuming to find or develop suitable problems.

   2. Symbolic systems lack what mathematicians would like them to do to help achieve educational goals. What properties are required to enhance the students experience and enjoyment of mathematics in your courses?

A session of the 1985 Canadian Mathematics Education Study Group explored how a symbolic mathematical system would impact and enrich the introductory calculus curriculum. The report of this group -- including a collection of individual experiences -- is available as part of the Proceedings of the 1985 CMESG Annual Meeting, deposited on the ERIC system (see [CMESG 85]).

We hope that the following extensive annotated bibliography will be of some use to you.

ASSMUS 85

E.F. Assmus, Jr., "Pi." Amer. Math. Monthly 92 (1985) 213-214.
How much from the standard "techniques of integration" chapter should be retained, now that symbolic antidifferentiation is readily avaible on home computers? This paper presents an example of a "conceptual problem" (the existence of π) showing the importance of change of variables and integration by parts in studying integration.

BUCHBERGER 83

B. Buchberger, G.E. Collins and R. Loos, eds., Computer Algebra: Symbolic and Algebraic Computation. Springer-Verlag, 1983. (2nd ed.)
First edition issued as a Supplementum to the journal Computing (1982). A basic book containing sixteen survey articles (with extensive references) about the theory and implementation of symbolic mathematical systems (the so-called "computer algebra").

CMESG 85

B.R. Hodgson and E. Muller, "Impact of symbolic manipulation software on the teaching of calculus." (Report of a working group). In Proceedings of the 1985 Annual Meeting of the Canadian Mathematics Education Study Group, C. Verhille, ed. (Available from Educational Resources Information Centre, Ohio State University, Columbus, Ohio.)
Includes contributions from participants to this working group, some of which are related to general issues while others deal with individual experiences.

COMP & MATH 84

J.T. Fey, ed., Computing and Mathematics: The Impact on Secondary School Curricula. NCTM, 1984.
Report of a conference sponsored by NSF. Of particular interest are the chapters "Impact of computing on algebra" and "Impact of computing on calculus".

COSERS 80

B.W. Arden, ed., What can be automated? MIT Press, 1980.
The Computer Science and Engineering Research Study. A huge report (nearly 1000 pages!) on all aspects of computer science. Pages 513-526 give a short introduction to algebraic manipulation.

COXFORD 85

A. Coxford, "School algebra: what is still fundamental and what is not." In [NCTM.YB 85] pp. 53-64.
"The push to incorporate symbolic mathematical systems in algebra is questionable because we are not sure of the relationships between procedural knowledge and skill and the understanding of algebra.
(...) I predict that more procedural knowledge will be needed to learn algebra than many would believe."

DAVENPORT 81

J.H. Davenport, "Effective mathematics - the computer algebra viewpoint." In Constructive Mathematics, F. Richman, ed. Springer-Verlag, 1981, pp. 31-43. (Lect. Notes in Maths, no. 873.)
An introduction to the theory underlying symbolic mathematical systems.

DAVENPORT 86 — J.H. Davenport, "Mathematics of computer algebra systems." In [ICMI 86] pp. 76-84.
This paper addresses the following question: what is the mathematics underlying symbolic mathematical systems and how does it relate to current curricula?

FATEMAN 80 — R.J. Fateman, "Symbolic and algebraic computer programming systems." Proc. ICME-IV (1980) Birkaüser, 1983. pp. 606-612.
A mini-course on symbolic and algebraic computer programming systems.

FATEMAN 85 — R.J. Fateman, "Comments on SMP." SIGSAM Bulletin 19 (3) (1985) 5-7.
Sharp criticism of one of the major currently available symbolic systems.

FEY & GOOD 85 — J.T. Fey and R.A. Good, "Rethinking the sequence and priorities of high school mathematics curricula." In [NCTM.YB 85] pp. 49-52.
"A small number of familiar and powerful mathematical ideas are at the heart of most common applications (...) A student assisted by [a symbolic mathematical system] need not endure a long skill-building apprenticeship in order to become an effective problem-solver - if the key organizing concepts are well understood."

HEID 83 — M.K. Heid, "Calculus with muMATH: implications for curriculum reform." Computing Teacher 11 (1983) 46-49.
A condensed version of some issues discussed in [COMP & MATH 84].

HODGSON & AL. 85 — B.R. Hodgson, E. Muller, J. Poland and P. Taylor, "Introductory calculus in 1990." In [STRASBOURG 85] pp. 255-258.
"We propose ways in which the introductory Calculus curriculum might respond to the recent and rapidly changing computer resources." Discussion stresses the use of a contextual approach, the qualitative analysis of functions in mathematical modelling and an interactive mode of classroom teaching.

HODGSON & POLAND 83 — B.R. Hodgson and J. Poland, "Revamping the mathematics curriculum: the influence of computers." CMS Notes 15(8) (1983) 17-23.
Outcome of working groups at the CMESG meetings of 1982 and 1983. Raises the question of the relevance, in the context of the actual computer revolution, of mathematics courses taught in the traditional form. Proposes scenarios of reasonable solutions to the changes needed in undergraduate mathematics education.

HOSACK.1 85 — J.M. Hosack, "The effect of computer algebra systems on the curriculum." Preprint, Colby College (Waterville, ME), 1985. (5 pages)
A general discussion regarding the use of symbolic mathematical systems in early courses (see also [LANE 85] for a presentation of the Colby Curriculum Project).

HOSACK.2 85 — J.M. Hosack, "A guide to computer algebra systems." Preprint, Colby College (Waterville, ME), 1985. (15 pages)
A comparison of the capacities of MACSYMA, Maple, muMATH, REDUCE and SMP.

ICMI 84

"The influence of computers and informatics on mathematics and its teaching." (An ICMI discussion document). L'enseignement mathématique 30 (1984) 159-172.
The discussion document prepared for the ICMI Symposium held in Strasbourg in March 1985 (see also [ICMI 86] and [STRASBOURG 85]).

ICMI 86

A.G. Howson and J.-P. Kahane, eds., The Influence of Computers and Informatics on Mathematics and its Teaching. (ICMI Study Series) Cambridge University Press, 1986.
The Proceedings of the ICMI Symposium held in Strasbourg in March 1985 (see also [STRASBOURG 85]). Contains an expanded version of [ICMI 84] as well as a selection of papers submitted to Strasbourg or written by invitation following the meeting. Of particular interest to symbolic systems are [DAVENPORT 86] and [LANE & AL. 86]. (A lively report of the presentations and discussions at the Strasbourg Symposium can be found in a paper by R. Biehler, R. Strässer and B. Winkelmann, Zentralblatt für Didaktik der Mathematik 18 (2) (1986).)

KENELLY & AL. 85

J. Kenelly, P. Henry and C.O. Jones, "The advanced placement program in calculus." In [NCTM.YB 85] pp. 166-176.
Some of the topics of the maths curriculum should not be treated with the computer. The authors make a parallel with machine translation of natural languages: "Here, the computer is very capable with mechanical substitutions but the rich subtleties are lost."

KUNKLE & BURCH 84

D. Kunkle and C.I. Burch, "Symbolic computer algebra: the classroom computer takes a quantum jump." Mathematics Teacher 77 (1984) 209-214.
Illustrates the use of muMATH for finding the sum of j^k (j = 1,...,N) for different values of k.

LANE 85

K.D. Lane, "Symbolic manipulators and the teaching of college mathematics: some possible consequences and opportunities." Preprint, Colby College (Waterville, ME), 1985. (13 pages)
Description of the Colby Curriculum Project integrating symbolic mathematical systems in the college curriculum. A condensed version appears in [MAA. PANEL 84].

LANE & AL. 86

K.D. Lane, A. Ollongren and D.R. Stoutemyer, "Computer-based symbolic mathematics for discovery." In [ICMI 86] pp. 133-146.
Examples illustrating the process of discovery in the context of symbolic mathematical systems.

MAA. PANEL 84

M.J. Siegel, ed., Panel on Discrete Mathematics in the First Two Years (Preliminary Report). MAA, Nov. 1984.
Included is a short "Report on the use of symbolic mathematics system in undergraduate instruction" by J. Hosack, K. Lane and D. Small. (The final report from the MAA Committee on Discrete Mathematics in the First Two Years was recently issued.)

MAPLE 84

B.W. Char, K.O. Geddes and G.H. Gonnet, "An introduction to Maple: sample interactive session." Research Report CS-84-04, Department of Computer Science, University of Waterloo. (15 pages)
An introduction to what can be done with a symbolic mathematical system, using the system Maple currently under development at the University of Waterloo.

MOSES 71

J. Moses, "Algebraic simplification: a guide for the perplexed." Comm. ACM 14 (1971) 527-537. "Symbolic integration: the stormy decade." Comm. ACM 14 (1971) 548-560.
Two papers from the Second Symposium on Symbolic and Algebraic Manipulation. These excellent expositions, although somewhat dated, give a lot of informations about the way computers can manipulate symbolic expressions and find antiderivatives. A good preparation for the reading of the more technical [BUCHBERGER 83].

NCTM.CONF 84

M.K. Corbitt and J.T. Fey, The Impact of Computing Technology on School Mathematics. (Report of an NCTM conference). NCTM, 1984. (5 pages)
A brief report from a conference held in March 1984. Includes recommandations related to the impact of computing technology on curriculum, instruction and teacher education.

NCTM.YB 84

J. Fey and M.K. Heid, "Imperatives and possibilities for new curricula in secondary school mathematics." In Computers and Education (1984 Yearbook), V.P. Hansen and M.J. Zweng, eds. NCTM, 1984. pp. 20-29.
Similar in spirit to [HEID 83] or [COMP & MATH 84]. Stresses "topics of diminished importance" and "topics of continued importance". (The whole 1984 Yearbook contains 27 papers divided in five parts: Issues; The computer as a teaching aid; Teaching mathematics through programming; Diagnostic uses of the computer; Bibliography.)

NCTM.YB 85

C.R. Hirsch and M.J. Zweng, eds., The Secondary School Mathematics Curriculum (1985 Yearbook). NCTM, 1985.
Of special interest to symbolic computations are the papers [COXFORD 85], [FEY & GOOD 85], [KENELLY & AL. 85] and [RALSTON 85].

NORMAN 83

A.C. Norman, "Algebraic manipulation." In Encyclopedia of Computer Science and Engineering, A. Ralston et al., eds. Van Nostrand, 1983. pp. 41-50.
A quick overview of different symbolic manipulation systems.

PAVELLE 85

R. Pavelle, ed. Applications of Computer Algebra. Reidel, 1985.
Based on a symposium organised by the American Chemical Society's Division of Computers in Chemistry and entitled "Symbolic algebraic manipulation in scientific computation".

POLAND 84

J. Poland, "Computers and the impending revolution in mathematics education." Ont. Math. Gaz., 23(2) (1984) 26-29.
A fresh discussion of some of the issues raised by the presence of computers and symbolic manipulation systems.

RALSTON 85

A. Ralston, "The really new college mathematics and its impact on the high school curriculum." In [NCTM.YB 85] pp. 29-42.
What changes should occur in the high school curriculum as a result of changes in the college curriculum (vg the role of discrete mathematics) and the direct impact of computers technology (vg symbolic mathematical systems) on the high school curriculum.

RAND 84

R.H. Rand, Computer Algebra in Applied Mathematics: An Introduction to MACSYMA. Pitman, 1984.
An introduction to the use of symbolic manipulation systems (viz. MACSYMA) in higher maths. ("This book is aimed at a reader who has had at least three years of college level calculus and differential equations.") The syntax of MACSYMA is learned while working some examples. Contains a few exercises with detailed solutions. "It is expected that it will not be long before computer algebra is as common to an engineering student as the now obsolete slide rule once was."

SCI. AMER. 81

R. Pavelle, M. Rothstein and J. Fitch, "Computer Algebra." Scient. Amer., Dec. 1981, pp. 136-152 (Version française: "L'algèbre informatique." Pour la science, fév. 1982, pp. 90-98.)
A most influential paper in making symbolic manipulation systems known to the general (scientific) public.

SIGSAM 85

SIGSAM Bulletin 18(4) and 19(1) (1984-85).
A special issue of the Bulletin of the "Special Interest Group on Symbolic & Algebraic Manipulation" (SIGSAM) of the ACM. Contains papers from the session on "Symbolic mathematical systems and their effects on the curriculum" held at ICME-5, Adelaide, 1984. Sixty-two pages of interesting reading. Some of the papers report on experiments done in high school or university.

SQUIRE 84

W. Squire, "muMATH system effective tool for algebra." SIAM News, Nov. 1984, p.4.
"The situation may be described as a potential revolution waiting for a textbook."

STEEN 81

L.A. Steen, "Computer calculus." Science News, 119 (1981) 250-251.
A short presentation of symbolic mathematical systems.

STEWART 84

I. Stewart, Review of [BUCHBERGER 83] Math. Intell., 6(1) (1984) 72-74.
Some comments on the general question: Will the computer, with its symbolic manipulation capability, put all mathematicians out of business?

STOUT 79

D.R. Stoutemyer, "Computer symbolic math and education: a radical proposal." SIGSAM Bulletin, 13 (2) (1979) 8-24.
An interesting discussion of the use of symbolic manipulation systems in the teaching of mathematics.

A revised and augmented version of this paper has appeared in [SIGSAM 85], pp. 40-53, under the title: "A radical proposal for computer algebra in education".

STOUT 83 — D.R. Stoutemyer, "Nonnumeric computer applications to algebra, trigonometry and calculus." Two-Year Coll. Math. J., 14 (1983) 233-239.
A general introduction to symbolic manipulation systems. Mentions some applications in abstract algebra.

STOUT 85 — D.R. Stoutemyer, "Using computer symbolic math for learning by discovery." In [STRASBOURG 85], pp. 185-190.
Some nice suggestions of projects using computer algebra for math discovery. See also [LANE & AL. 86].

STRASBOURG 85 — The Influence of Computers and Informatics on Mathematics and its Teaching. Supporting papers for the Symposium organised by ICMI. Strasbourg, March 1985. (321 pages)
The papers presented by the participants to the ICMI Symposium, many of which have to do with symbolic systems. Copies of this monograph can be ordered from the MAA or from the IREM, Université Louis-Pasteur, 67084 Strasbourg Cédex, France.

TALL & WEST 86 — D. Tall and B. West, "Graphical insight into calculus and differential equations." In [ICMI 86] pp. 107-119.
Use of symbolic mathematical systems can lead to an interplay with graphical systems. This paper reports on the development of interactive high resolution graphics approaches at different levels of teaching calculus and differential equations. See also, by the first author, "Continous mathematics and discrete computing are complementary, not alternatives", Coll. Math. J. 15 (1984) 389-391, "Understanding the calculus", Math. Teaching No. 110 (March 1985) pp. 49-53 and "Visualizing calculus concepts using a computer", in [STRASBOURG 85] pp. 291-295.

USISKIN 84 — Z. Usiskin, "Mathematics is getting easier." Math. Teacher 77 (1984) 82-83.
"Some skills are clearly necessary, but (...) too much else should be learned about mathematics to waste time in practicing obsolete skills. Mathematics is getting easier [with muMATH]. We will not be able to keep this secret from our students forever."

WILF 82 — H.S. Wilf, "The disk with the college education." Amer. Math. Monthly, 89 (1982) 4-8.
muMATH is coming! muMATH is coming! A paper intended as a "distant early-warning signal" for the mathematical community.

WILF 83 — H.S. Wilf, "Symbolic manipulation and algorithms in the curriculum of the first two years." In The Future of College Mathematics, A. Ralston and G.S. Young, eds. Springer-Verlag, 1983, pp. 27-40.
Expands on the issues raised in [WILF 82]. "It can be very unsettling to realize that what we previously thought was a very human ability (...) can actually be better done by "machines"." (Also contains a description of a second semester sophomore course introducing algorithms.)

WINKEL 84

B. Winkelmann, "The impact of the computer on the teaching of analysis." Int. J. Math. Educ. Sci. Techn. 15 (1984) 675-689.
A basic discussion of the ways the computer capabilities (among others, the symbolic capabilities) will influence the teaching of calculus.

YUN & STOUT 80

D.Y. Yun and D.R. Stoutemyer, "Symbolic mathematical computation." In Encyclopedia of Computer Science and Technology, J. Belzer et al., eds. M. Dekker, 1980. Vol. 15, pp. 235-310.
A general discussion of symbolic manipulation systems. Includes a guide to some existing systems and a discussion of basic issues and alternatives for building up such a system. The last 30 pages are devoted to applications: algebra, nonscalar analysis, numerical analysis, celestial mechanics, general relativity, high-energy physics.

VISITING MATHEMATICIANS

Name	*Home*	*Speciality*	*Dates*
		Acadia University	
Butcher, J.	New Zealand	Numerical Analysis	5/86-6/86
Krapez, A.	Yugoslavia	Algebraic Func. Equations	6/86-6/87
		Queen's University	
Alaoui Ismaili, A. A.	Morocco	Operations Research	9-12/86
Bogolyubov, N.N.	U.S.S.R.	Mathematical Physics	9-12/86
Dilcher, K	Germany	Algebra	9-12/86
Orecchia, F.	Italy	Algebraic Geometry	10/86
Roitman, M.	Israel	Algebra	-10/96
Sun, J.	China	Num. Analysis & Applied Math	9/86-8/87
Wang, T.	China	Applications of Statistics	9/86-8/87
Soublin, J.P.	France	Number Theory,Algebra	9/86-10/86
Zine, A.	Morocco	Numerical Analysis	9/86-12
		University of Calgary	
Schmidt, E.T.	Hungary	Lattice Theory	9/86-4/87
		University of Western Ontario	
Baur, G.	West Germany	Analysis	1/87-3/87
Eastham, M.S.P.	england	Applied Mathematics	9/86-12/86
Frohlich, A.	England	Number Theory	8/86-10/86
Kratz, W.	West Germany	Analysis	7/86-9/86
Miransky, V.A.	U.S.S.R.	Theoretical Physics	9/86-12/86
		York University	
Kato, A.	Japan	Theoretical Topology	9/86-5/87
Morisugi, K.	Japan	Algebraic Topology	8/86-12/86
Rosicky, J.	Czechoslavakia	Category Theory	1/87-4/87
Szusz, P.	U.S.A.	Analysis	11/86