

Using Orientation Statistics to Investigate Variations in Human Kinematics

by

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Summary

This paper applies orientation statistics to investigate variations in upper limb pose of human subjects drilling at 6 different locations on a vertical panel (cf. Figure 2). Some of the drilling locations are kinematically equivalent in the sense that the same pose could be used for these locations. Upper limb pose is measured by recording the coordinates of 4 markers attached to the subject hand, forearm, arm, and torso (cf. Figure 1). A 3×3 rotation characterizes the relative orientation of one body segment with respect to another. Replicates are available since each subject drilled at the same location 5 times. Upper limb poses for the 6 drilling locations are compared with one-way analysis of variance tests for rotations. These tests rely on tangent space approximations at the estimated modal rotation of the sample. A parametrisation of rotations in terms of unit quaternions simplifies the computations. The analysis detects significant differences in pose between all pairs of drilling locations. The smallest changes, less than 10 degrees at all joints, are obtained for the kinematically equivalent pairs of locations. A short discussion of the biomechanical interpretation of these findings is presented.

Keywords: Directional data, Drilling task, Joint kinematics, Multivariate statistics, Quaternions, Rotations

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1 Introduction

The analysis of human movement has been the object of several studies in the past. An interesting review of the various domains of research on human kinematics is presented in Woltring (1989). Several companies have marketed camera systems to automatically record spatial motion of markers, affixed to the body, using different technologies e.g. Optotrack, Selspot, Peak, Vicon, Elite, Polhemus, MacReflex, and others. The data from these high performance recording systems permit the calculation of time varying human kinematics such as position, velocity, and acceleration. For some systems, markers consist of one landmark whose time varying coordinates are recorded in the system reference frame (see Olshen et al, 1989). The one landmark markers do not permit a complete characterization of the orientation of the body segment to which they are attached; at least three landmarks on each body segment must be used to characterize orientation. For instance, a 4 landmark marker, as shown in Figure 1, can be used to characterize the orientation of any of the upper limb body segments. The knowledge of the relative orientations of adjacent limb segments provides a mean to characterize the limb geometric configuration i.e. limb pose.

Characterization of limb pose is helpful for conducting motor control studies in various biomedical research fields: neurophysiology, rehabilitation, sport performance, etc. For instance, by analyzing limb motion, one may be able to infer on the structure of the neural system that supervises the control of the limb (e.g. see Sabes et al., 1997). One may also want to use upper limb kinematic data to investigate different optimization schemes underlying the control of limb motion e.g. minimization of the torque-change criterion for a given limb movement (Kawato et al., 1990). It is thought that such optimization is necessary to determine the particular pose

that a human subject would choose to perform a task. As a matter of fact, since the upper limb is a redundant manipulator, there is an infinite number of allowable limb poses to perform hand interactive tasks. The upper limb alone has at least seven degrees of freedom: three dof at the shoulder, one dof at the elbow and three at the wrist (McCarthy 1990, chapter 5). This additional dof allows a human subject to modify its upper limb pose while keeping the same hand position and orientation relative to the environment.

The experiment considered in this paper was conducted in order to investigate whether such optimization scheme exists in the context of drilling operation. This particular task was chosen because of the interests of one author in investigating human motor control in tool usage. Repeated choice of a particular pose over the infinite number of possibilities may indicate that limb pose could be chosen to fulfill some specific criterion. For instance, subjects could choose a more extended limb to maximize the force that can be produced at the hand in the axial direction of the drill. Several other criteria could be suggested but currently, no one knows if any such criterion exists. The drilling study actually attempted to investigate the existence of such a criterion by analyzing the consistency in upper limb pose of human subjects in a given task.

The data set for this experiment was recorded using markers represented in Figure 1. Each marker consists of four non-collinear IREDs (infrared emitting diodes) whose time varying coordinates are recorded by a camera system. At each time point the coordinates of the four landmarks of a marker can be used to calculate a 3×3 rotation matrix characterizing its orientation. The relative orientation of two human body segments is obtained by computing the relative orientation of two rigid markers, assuming that each marker is rigidly fixed on one of the segments.

Insert Figure 1 about here

Despite recent advances in computing human kinematics, one aspect is still poorly covered, that of providing the end-user with adequate statistical tools to analyze experimental data. Translations of markers in space can be analyzed with standard statistical methods since these are linear variables. The situation is quite different when one is interested in statistical methods for investigating the relative rotations between two adjacent body segments. This problem has not really been addressed in the biomechanical literature yet. Veldpaus et al. (1988), De Lange et al. (1990), and Woltring (1994) have suggested to evaluate the stability of an estimated orientation by sensitivity analysis; however statistical tests and inference procedures are not presented in any of these articles.

This paper introduces tests for comparing the relative orientation of markers using techniques of orientation statistics. Down (1972), Khatri and Mardia (1977), and Prentice (1986) have contributed to this subject; see also the review paper of Jupp and Mardia (1989). Commonly, in human kinematics studies, the sample size is small and rotations are clustered together. Thus the so-called tangent space techniques, introduced by Downs (1972), are suited for human kinematics. Substantial computational savings are obtained by working with the quaternion representations of rotations, as proposed by Prentice (1986).

Section 2 of the paper builds on Downs (1972) to construct one-way analysis of variance tests for clustered samples of rotations. It uses a residual vector, characterizing the discrepancy between a rotation in a sample and the sample mean rotation, to construct statistical tests. Standard multivariate normal procedures are shown to apply to the tangent space model for rotations presented in Section 2. Section 3 suggests a computationally efficient method, based on quaternions, to calculate the

statistics of Section 2. Section 4 applies this methodology to the drilling data and suggests techniques to interpret differences between several samples of rotations. It shows that there does not appear to exist a criterion for selecting drill poses since most experimental subjects had statistically different poses for the drilling locations that were kinematically equivalent.

2 One-way analysis of variance for rotations

2.1 Random rotations

A rotation is an orthogonal matrix whose determinant is equal to 1. The set of rotations for \mathfrak{R}^3 -vectors is denoted $SO(3)$. The matrix Fisher-von Mises distribution introduced by Downs (1972), see also Khatri and Mardia (1977), is a general exponential family model for describing the scatter of random orthogonal matrices around their modal value. Prentice (1986) shows how to reparametrise this model for data points that are rotations. In postural kinematics studies, the random rotations are closely clustered around their modal value. This means that the experimental errors are small and that tangent space techniques apply. Downs' model (1972) specifies that the errors, in the tangent space, have a trivariate normal distribution.

If M is an $SO(3)$ -rotation, the tangent space to $SO(3)$ at M is the 3-dimensional vector space of the 3×3 skew-symmetric matrices. This means that

$$M\{I + \Phi(a)\}$$

describes the rotations around M when a varies in a neighborhood of \mathfrak{R}^3 origin,

where the function $\Phi(\cdot)$ maps an \mathfrak{R}^3 -vector a into the skew symmetric matrix,

$$\Phi(a) = \begin{pmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{pmatrix}. \quad (2.1)$$

It is clear that $\Phi(\cdot)$ is an invertible mapping. If V is a 3×3 skew-symmetric matrix then $a = \Phi^{-1}(V) = (V_{32}, V_{13}, V_{21})^t$. Unfortunately, $M\{I + \Phi(a)\}$ is not a rotation. The exponential map provides a more satisfactory description of neighborhoods of M . One can show that

$$\begin{aligned} \exp\{\Phi(a)\} &= I + \Phi(a) + \frac{1}{2!}\{\Phi(a)\}^2 + \frac{1}{3!}\{\Phi(a)\}^3 + \dots \\ &= \cos(\theta)I + \frac{\sin(\theta)}{\theta}\Phi(a) + \frac{1 - \cos(\theta)}{\theta^2}aa^t. \end{aligned} \quad (2.2)$$

is a rotation of angle $\theta = (a^t a)^{1/2}$ around axis $a/(a^t a)^{1/2}$. This means that the formula

$$M \exp\{\Phi(a)\}$$

describes, when a is close to \mathfrak{R}^3 's origin, rotations that are closed to M . Chapter 2 of McCarthy (1990) provides an elementary presentation of the tangent space and exponential map for $SO(3)$ -rotations.

Let M denote a fixed 3×3 rotation representing the true orientation of a marker. The rotation X recorded for this marker is not usually equal to M . A simple model for describing the scatter of a sample of experimental rotations around M is given next.

2.2 Estimation of the modal rotation

Downs' (1972) statistical model for a sample of n rotations clustered around their modal value can be expressed as

$$X_i = M \exp\{\Phi(\varepsilon_i)\}, \quad i = 1, \dots, n, \quad (2.3)$$

where the ε_i 's are independent copies of a trivariate normal random vector ε whose components are small or $O_p(\kappa^{-1/2})$. Here κ indexes the magnitude of the error, large κ 's give small errors. The tangent space inference relevant to kinematics studies is obtained by letting κ go to infinity. The section reviews the sampling properties of Downs' (1972) estimator for M and suggests a formula for estimating residuals in (2.3).

Implicit in Downs' (1972) is the idea that the maximum likelihood estimator for M under a Fisher-von Mises matrix distribution is the rotation matrix M that is closest to $\bar{X} = \sum X_i/n$ in the least squares sense; i. e. such that $\text{tr}(\bar{X} - M)^t(\bar{X} - M)$ is minimum. Thus, the maximum likelihood estimator of M maximizes $\text{tr}(M^t \bar{X})$. This maximization problem occurs in spherical regression (Stephens, 1979, Chang, 1993) and in Procrustes analysis (Sibson, 1978). Its solution is based on the singular value decomposition of \bar{X} ,

$$\bar{X} = \hat{P} \text{diag}(\hat{\gamma}_1, \hat{\gamma}_2, \hat{\gamma}_3) \hat{Q}^t \quad (2.4)$$

where \hat{P} and \hat{Q} are rotations and $\hat{\gamma}_1 > \hat{\gamma}_2 > |\hat{\gamma}_3|$ are the singular values of \bar{X} . One has $\hat{M} = \hat{P} \hat{Q}^t$; \hat{M} is called the sample mean rotation, or the mean rotation, throughout this paper. Inference procedures for M are constructed using residual vectors as building blocks, see Jupp (1988) for a discussion of residuals for directional data.

The residual for rotation X_i in model (2.3) can be defined by

$$r_i = \Phi^{-1} \left(\frac{\hat{M}^t X_i - X_i^t \hat{M}}{2} \right). \quad (2.5)$$

for $i = 1, \dots, n$. Note that the three elements of r_i are less than 1 in absolute value. This vector characterizes the discrepancy between X_i and \hat{M} ; $r_i = 0$ when $X_i = \hat{M}$. One also has $\bar{r} = \Phi^{-1}\{(\hat{M}^t \bar{X} - \bar{X}^t \hat{M})/2\} = 0$, since from the construction of \hat{M} using (2.4), $\hat{M}^t \bar{X}$ is a symmetric matrix.

The next proposition whose proof is given in the Appendix describes the first order propagation of the sampling errors ε_i to the \hat{M} and r_i .

Proposition 1

When in model (2.3) the ε_i 's have components which are $O_p(\kappa^{-1/2})$, one has

- i) $\hat{M} = M \exp\{\Phi(\bar{\varepsilon})\} + O_p(\kappa^{-1})$;
- ii) $r_i = \varepsilon_i - \bar{\varepsilon} + O_p(\kappa^{-1})$.

From the proof of Proposition 1 one has the following representation, $\text{tr}(\hat{M}^t \bar{X}) = 3 - \sum(r_i^t r_i)/n + o_p(\kappa^{-1})$.

Proposition 1 shows that, in concentrated samples, estimating the mean rotation is analogous to estimating the mean in a trivariate sample. When the experimental errors are normally distributed, a $100(1-\alpha)\%$ confidence region for the unknown modal rotation M is obtained from Hotelling's distribution. It is given by

$$\left\{ \hat{M} \exp\{\Phi(a)\} : \frac{n-3}{(n-1)3} a^t S^{-1} a < F_{3, n-3, \alpha} \right\}, \quad (2.6)$$

where $S = \sum r_i r_i^t / (n-1)$ is the residual variance-covariance matrix. Confidence regions for the rotation axis and for the rotation angle can be derived from Rivest (1989).

2.3 Statistical methods for comparing the modal rotations of several samples

This section reviews standard methods of multivariate statistics that, in view of Proposition 1, apply to the comparison of samples of rotations. Let $\{X_{ij} : j = i, \dots, n_i \quad i = 1, \dots, \mathcal{I}\}$ represent \mathcal{I} samples of rotations of sizes $\{n_1, n_2, \dots, n_{\mathcal{I}}\}$. The statistical model for this data set is

$$X_{ij} = M_i \exp\{\Phi(\varepsilon_{ij})\}, \quad j = i, \dots, n_i \quad i = 1, \dots, \mathcal{I} \quad (2.7)$$

where M_i is the modal rotation for the i th sample and the ε_{ij} 's are independent random vectors distributed according to a $N_3(0, \Sigma)$ distribution, and $\text{tr}(\Sigma)$ is small or $O(\kappa^{-1})$. This section suggests test statistics for the hypothesis $H_0 : M_1 = M_2 = \dots = M_{\mathcal{I}}$ versus the alternative $H_a : M_i \neq M_j$ for at least one pair (i, j) .

For each sample, one calculates the mean rotation \hat{M}_i and the residuals r_{ij} defined in Section 2.2. In view of ii) of Proposition 1, an estimator for the common variance-covariance matrix Σ is given by $S = \sum r_{ij} r_{ij}^t / (\sum n_i - \mathcal{I})$. Furthermore, the multivariate normality of the errors ε_{ij} can be assessed with Mardia's multivariate skewness and kurtosis coefficients (Rencher, 1995) calculated on the residuals \hat{r}_{ij} .

Testing H_0 , the hypothesis of equality of the mean rotations, can be regarded as a repeated measure problem where each rotation contributes its three dimensional vector of errors ε . The test statistic for H_0 depends on the variance-covariance matrix of ε_{ij} 's. If it is spherical, that is $\Sigma = cI$ for some positive c , then a simple test statistic for H_0 is given by

$$F_{obs} = \frac{\{\sum_i n_i \text{tr}(\hat{M}_i^t \bar{X}_{i.}) - \sum_i n_i \text{tr}(\hat{M}^t \bar{X}_{..})\} / \{3(\mathcal{I} - 1)\}}{\{3 \sum_i n_i - \sum_i n_i \text{tr}(\hat{M}_i^t \bar{X}_{i.})\} / (3 \sum_i n_i - 3\mathcal{I})}, \quad (2.8)$$

where \hat{M} denotes the mean rotation calculated on the pooled sample of size $\sum n_i$. Its null distribution is F with $3(\mathcal{I} - 1)$ and $3 \sum_i n_i - 3\mathcal{I}$ degrees of freedom. This is

a generalization of Watson and Williams (1956) statistic for the homogeneity of the mean directions.

It is well known that (2.8) is liberal when the sphericity assumption is violated (Crowder and Hand, 1990, chapter 3). Mauchly's sphericity test rejects this Hypothesis when

$$\chi_{obs}^2 = - \left(\sum_i n_i - \mathcal{I} - \frac{23}{18} \right) \log \left\{ \frac{\det(S)}{\{\text{tr}(S)/3\}^3} \right\},$$

is large, where $\det(A)$ denotes the determinant of A and S is the pooled variance covariance matrix. Its null distribution is, in large samples, the χ^2 on 5 degrees of freedom. Critical values for small samples are available (Kres, 1983). Using the sphericity test in directional statistics had been advocated by Rivest (1986).

When the sphericity hypothesis is rejected, Hypothesis H_0 can be tested with Wilks' lambda statistic. It is written in terms of \mathbf{E} , the dispersion matrix for the errors, $\mathbf{E} = \sum r_{ij} r_{ij}^t$; and $\mathbf{E} + \mathbf{H}$, the dispersion matrix of the residuals $\{r_{ij}^0\}$ around a common mean rotation, \hat{M} , for all samples combined. One has $\mathbf{E} + \mathbf{H} = \sum r_{ij}^0 r_{ij}^{0t}$. Wilks' test statistic rejects the null hypothesis when

$$\Lambda = \det(\mathbf{E})/\det(\mathbf{E} + \mathbf{H}) \tag{2.9}$$

is small. We use Rao's F-approximation to the distribution of (2.9) to calculate p-values (see Rencher, 1995, p.182). If $t = \{(9\mathcal{I}^2 - 18\mathcal{I} + 5)/(\mathcal{I}^2 - 2\mathcal{I} + 5)\}^{1/2}$, $df_1 = 3(\mathcal{I} - 1)$ and $df_2 = t\{\sum n_i - (\mathcal{I} + 5)/2\} - (3\mathcal{I} - 5)/2$, then

$$\frac{(1 - \Lambda^{1/t}) df_2}{\Lambda^{1/t} df_1}$$

is approximately distributed, under the hypothesis of homogeneity, as an F-distribution with df_1 and df_2 degrees of freedom.

Sample sizes	Individual variance c								
	10^{-4}	10^{-2}	0.1	0.5	1.0	1.5	2	3	4
5,5,5	4.3	5	3.4	1.9	3.5	3.8	2.5	3.6	2.5
5,5,10	5	4	3.4	1.5	4.5	4.2	3.7	4	3.7
5,10,15	4.5	5.5	3	1.9	5	6.2	4	5.3	5.4
20,20,20	4.8	4.7	2.9	2	7.2	11.5	6.7	6.8	5.7
30,30,30	5.3	4.5	3.1	2.5	6.4	13	10.2	8.4	6.7

Table 1: Empirical levels, expressed in percentage, for Wilks' test with a nominal level of 5% when the critical value is obtained with Rao's F-approximation.

Test statistics should be invariant to a change from $\{X_{ij}\}$ to $\{X_{ij}^t\}$. This holds true for (2.8). This is not exactly true for Mauchly's and Wilks' tests. For these two tests there are, under the null hypotheses, small, or $o_p(\kappa^{-1})$, differences between the statistics calculated on $\{X_{ij}\}$ and those obtained from $\{X_{ij}^t\}$.

2.4 Simulation results

To investigate the quality of the tangent space approximations to the distribution of Wilks' lambda statistics (2.9), data were simulated from (2.7) with $\mathcal{I} = 3$, for various samples sizes. Empirical levels corresponding to a nominal level of 5% were calculated from 2000 Monte Carlo replications. The standard error associated with the estimated percentages is 0.5%. In the simulations, $\Sigma = cI$ for various values of c . The results appear in Table 1. For $c \leq .01$ the experimental p-values are equal to the nominal 5%, up to experimental errors. For $.01 < c \leq 0.5$ the experimental p-values are smaller than or equal to 5%. The proposed version of Wilks' test is conservative. This suggests that the large concentration approximations of this section are reliable

provided that the largest eigenvalue of the variance-covariance matrix Σ is less than 0.5.

3 A quaternion implementation of the analysis

The quaternion representation (Hamilton, 1969) of a rotation of angle θ around axis u , where u is a unit vector in \mathfrak{R}^3 , is a unit vector q in \mathfrak{R}^4 defined by $q = (\cos(\theta/2), \sin(\theta/2)u^t)^t$. Unit vectors q and $-q$ represent the same rotation. This is a 2 to 1 representation. Rooney (1977) provides a good introduction to this representation. Working with quaternions allows a substantial reduction in the computational effort required to calculate residuals r_i , defined in (2.5), and the test statistics of Section 2. Prentice (1986) introduced quaternions as a useful statistical tool for comparing samples of rotations.

Quaternions are endowed with a special product corresponding to rotation multiplication. If $p = (p_1, p_2, p_3, p_4)^t$ and $q = (q_1, q_2, q_3, q_4)^t$ are quaternion representations of rotations X_1 and X_2 respectively, then the dot product between p and q is given by:

$$p \cdot q = \begin{pmatrix} p_1 & -p_2 & -p_3 & -p_4 \\ p_2 & p_1 & -p_4 & p_3 \\ p_3 & p_4 & p_1 & -p_2 \\ p_4 & -p_3 & p_2 & p_1 \end{pmatrix} q = Pq. \quad (3.10)$$

In other words, $p \cdot q$ is the matrix product of P by q . Since P is a rotation, Pq is a unit vector. It can be shown that this unit vector is the quaternion representation of the matrix product of X_1 multiplied by X_2 . Also, $(p_1, -p_2, -p_3, -p_4)^t$ is the quaternion for the inverse of X_1 , X_1^t , and $P^t q$ is the quaternion representation of $X_1^t X_2$. Prentice (1986) gives further properties of quaternions.

Let $\{q_i : i = 1, \dots, n\}$ be a quaternion representation of the sample $\{X_i\}$. If p is the quaternion representation of rotation M , then

$$\text{tr}(M^t \bar{X}) = 4p^t \left(\frac{1}{n} \sum q_i q_i^t \right) p - 1.$$

This identity can be deduced from formula (2.2) of Prentice (1986). For the sake of completeness a short proof is given in the appendix. Thus the quaternion representation of the mean rotation \hat{M} is the eigenvector corresponding to the largest eigenvalue of $\sum q_i q_i^t$.

This suggests the following strategy for analyzing a sample of rotations. Let Q denote the $n \times 4$ matrix of the q_i^t 's. Take \hat{p} , the quaternion for the sample mean rotation, equal to the eigenvector corresponding to the largest eigenvalue of $Q^t Q$. Then rows of $R = Q \hat{P}$, where \hat{P} is constructed from \hat{p} as in (3.10), are the quaternions of the residual rotations $\hat{M}^t X_i$. As shown in the appendix, the residuals r_i defined in (2.5), can be derived easily from R , $r_i = 2R_{i1}(R_{i2}, R_{i3}, R_{i4})^t$. Quaternions allow substantial computational savings. All calculations of Section 4 are based on quaternions.

Prentice (1986) derived large sample tests and inference procedures for \hat{M} and the eigenvalues of $Q^t Q$. In terms of the notation of this section, the $100(1-\alpha)\%$ large sample confidence region for the sample mean rotation given by his formula (5.1), contains rotations $\hat{M} \exp\{\Phi(a)\}$ such that

$$na^t \hat{A} S^{-1} \hat{A} a < \chi_{3,\alpha}^2,$$

where $\hat{A} = ([R^t R]_{11} I - [R^t R]_{22})/n$, and $[R^t R]_{11}$ is the (1, 1) element of $R^t R$ while $[R^t R]_{22}$ is the 3×3 matrix obtained by deleting the first row and the first column of $R^t R$. In clustered samples, as κ goes to infinity, $\hat{A} = I + o_p(\kappa^{-1})$. Comparing this with the tangent space confidence region derived in Section 2 suggests a $100(1-\alpha)\%$

confidence region valid when the sample is either large or clustered. This region contains rotations $\hat{M} \exp\{\Phi(a)\}$ such that

$$\frac{n-3}{3(n-1)} a^t \hat{A} S^{-1} \hat{A} a < F_{3,n-3,\alpha}.$$

4 Analysis of upper limb pose

This section considers the analysis of upper limb pose variations in human subjects performing a drilling task. This study was performed in the Newman Laboratory for Biomechanics and Rehabilitation at MIT. The detailed description of the study can be found in Rancourt (1995). Limb pose was recorded by SELSPOT infrared cameras along with TRACK software created in the Newman Laboratory (Antonsson and Mann, 1989; Rowell and Mann, 1989). Briefly, one objective of the study was to determine whether upper limb pose chosen by human subjects in a drilling task was constant for apparently similar experimental situations. The interest in the drilling task was to investigate how drill operators deal with the redundancy issue of limb postural control. Characterization and statistical analysis of limb poses was therefore highly critical for the study. Statistical methods presented in the previous sections are well suited for this application.

4.1 Experimental design and data collection

Eight different SUBJECTS participated in the study. Each subject was instructed to drill at 6 different pre-determined LOCATIONS, on a vertically oriented metal plate, shown in Figure 2. Subjects were told to use only one hand to operate the drill, at any of the six locations for a period of 2-3 seconds. During that time period, the drilling upper limb pose was measured by a SELSPOT infrared camera system.

The limb pose was defined via four IRED markers affixed at different locations (cf Figure 1): the first one was rigidly connected to the subject’s back at the level of the seventh cervical vertebra, the second one to the drilling arm, the third to the drilling forearm, and the fourth to the drill itself. While the subject was drilling, the orientation and location of each four markers was recorded with respect to the laboratory reference frame. Each marker orientation was characterized by a 3×3 rotation matrix.

Insert Figure 2 about here

The upper limb pose was finally defined by computing the relative orientation of adjacent markers, thereby computing the relative orientation of two adjacent body segments. For instance, if X_b and X_a are rotations representing the orientations of the back and the arm respectively, then $X_b^t X_a$ represents the relative orientation of the arm with respect to the back. One should notice that because markers were not rigidly fixed to the bones, the relative orientation is not exactly equal to the rotations that occurred at the upper limb joints. The data set includes skin motion that perturbs sample rotations. Nevertheless, the data can still be used to investigate repeatability of limb pose if it is assumed that skin motion characteristics were stable all through the experiment. Special care was taken to ensure stability of the markers on the limb segments and thus, the assumption is tenable. Furthermore, data on two different subjects are not directly comparable since markers were not placed at exactly the same orientation on every subject.

Upper limb pose can be characterized by three relative rotations at the three upper limb joints: the shoulder, the elbow and the wrist. This defines the three level factor JOINT. Each subject performed 30 trials, 5 at each of the 6 LOCATIONS,

in a completely randomized design. Drilling LOCATIONS 1 to 4 were all positioned on the same vertical line. Drilling LOCATION 2 was set at the subject’s shoulder height. LOCATION 1 was set over LOCATION 2 at a distance equal to 20% of the subject’s upper limb length. LOCATION 3 was set below LOCATION 2 at a distance equal to 50% of the subject’s arm length, while LOCATION 4 was set directly below LOCATION 3 at the subject’s elbow height. LOCATIONS 5 and 6 were set at the same height as LOCATIONS 2 and 3 respectively, but 20 cm to the left of the vertical line. Each subject contributed 90=30 x 3 rotations to the data set. In the analysis, we considered that data set as 144 = 8 (SUBJECT) x 3 (JOINT) x 6 (LOCATION) samples of size 5. There were 129 non empty samples since, in some instances, one of the markers was out of the camera field of view. This happened when a subject took a position that prevented some of the markers to be registered properly by the camera system. As a matter of fact, a subject could significantly rotate his whole body about the drill tip, avoiding drill slip and without modifying its upper limb pose. Since limb relative orientations are investigated in this paper, we assume that the relative orientations for the empty and the non-empty samples are similar. We analyzed the 24 combinations of JOINT×SUBJECT separately.

4.2 Comparisons of the poses at the 6 drilling locations

Upper limb orientation analysis was performed by computing mean rotations for all non empty experimental samples. For each sample, the trace of the variance-covariance matrix, $\sum r_i^t r_i / 4$, was calculated. For the 129 samples under investigation, the trace ranged between .001 and .12 with a mean value of .013. Thus replicated observations of the same SUBJECT in the same POSITION did not exactly produce the same orientation. A trace of .013 means that the expected squared

angle, in radians, of the rotation needed to go from the sample mean rotation to a sample rotation is .013. Taking the square root and converting to degrees yields 6.5 degrees as the mean angle; this angle characterizes the variability within the average sample. This is a situation where large concentration asymptotics apply. To investigate whether in (2.3) the errors ε were normally distributed, we calculated the multivariate skewness and kurtosis coefficients on the residuals of the 129 samples. This led to two empirical distribution functions, calculated with 129 data points each, for skewness and kurtosis respectively. We compared these distributions to quantiles of the skewness and kurtosis distributions for trivariate normal samples of size 5. The corresponding QQ-plots are given in Figure 3. They show that the normality assumption is tenable.

Insert Figure 3 about here

For each null hypothesis of homogeneity to be tested, the sphericity test was calculated on the pooled variance-covariance matrix. More than half of these tests were significant at the 5% level. This led us to reject the hypothesis of sphericity and to use Wilks' statistic to test the homogeneity of the rotations. For each JOINT×SUBJECT combination POSITIONS 1 to 4 were compared with a 4-sample Wilks statistics. For each test the null hypothesis was rejected at the 10^{-3} level. We carried out pairwise tests of homogeneity for comparing 5 pairs of POSITIONS: 1-2, 2-3, 3-4, 2-5 and 3-6. One would expect to observe smaller differences for pairs 2-5 and 3-6 since these tasks are kinematically equivalent. All other tests should reveal differences in upper limb pose. For POSITION pairs 1-2, 2-3, and 3-4, 80% of the 65 tests were significant at the 5% level; this percentage dropped to 62.5% for pairs 2-5 and 3-6. More than 50% of the non significant tests came from two subjects. For the others, most of the tests were significant.

4.3 Understanding changes in drilling poses

To get a better appreciation of variations in limb pose, one can look at the orientation differences between pairs of POSITIONS. Thus, for each JOINT×SUBJECT combination, the angles of the rotation matrices for the changes in orientation for the 5 pairs of POSITIONS were calculated. These angles are invariant to changes in the position of the markers; thus, they characterize the movement that took place at the joint, including skin motion. To prove this, consider, $\hat{M}_1, \hat{M}_2, \dots, \hat{M}_6$, the 6 mean rotations for say the shoulder joint of individual 1. Rotating the back marker and the arm marker by rotations of R_b and R_a respectively would yield $R_b^t \hat{M}_1 R_a, R_b^t \hat{M}_2 R_a, \dots, R_b^t \hat{M}_6 R_a$ as mean rotations. The rotation for the changes in orientation, from POSITION 1 to POSITION 2 is $R_a^t \hat{M}_1^t \hat{M}_2 R_a$. Now if $\hat{M}_1^t \hat{M}_2$ is a rotation of angle $\hat{\theta}_{12}$ around axis \hat{u}_{12} , the $R_a^t \hat{M}_1^t \hat{M}_2 R_a$ is a rotation of angle $\hat{\theta}_{12}$ around axis $R_a^t \hat{u}_{12}$. In other words, the angle of the rotation for the change in orientation is invariant to marker displacement, as stated above. An analysis of variance of the 106 angles (14 were missing) revealed no interaction between SUBJECT and any other factors. Least squares means for the JOINT by change of POSITION classification are presented in Table 2. The least significant difference at a level of 5% is about six degrees.

Table 2 agrees with the hypothesis that changes in orientation at the three joints are generally smaller for the two pairs of kinematically equivalent positions, (2,5) and (3,6). The significance of such differences needs to be established in relation to the particular characteristics of the upper limb that is investigated. For instance, variations in the order of 8 to 10 degrees at each of the upper limb joints would probably not produce much difference in the mechanics of the upper limb, for the upper limb configurations that were selected.

	Pairs of positions				
Joint	1-2	2-3	3-4	2-5	3-6
wrist	17.37	18.24	18.65	8.17	8.83
elbow	12.46	7.36	12.62	9.76	7.27
shoulder	26.88	20.02	13.75	9.64	8.23

Table 2. Mean angles, Expressed in Degrees, for Changes in Orientation of Five Pairs of Positions at three Joints.

The analysis of the upper limb pose data presented in this section raises several statistical problems that cannot be addressed here. Table 2 summarizes the angles for the changes of orientation. It would also be interesting to investigate their axes. Does the axis for the change in orientation stay constant when going from POSITION 1 down to POSITION 4? New techniques need to be devised to assess the variability in the estimated axes. Another issue for investigation is the relationship between the orientations at the three JOINTs. The pose of a subject is characterized by three rotations, one at each JOINT. Investigating the dependencies between the three joints and finding meaningful biomechanical interpretations for possible relationships is another challenging research problem that is highly relevant to the understanding of human movement. The model proposed by Prentice (1989) is relevant to this research.

5 Discussion

The statistical approach presented in this paper was able to provide an objective evaluation to the consistency in limb pose for the drilling task. Statistical analysis

of upper limb pose suggests that pose, in a drilling task, would not be selected upon a selection criterion leading to a unique solution. Indeed, the variability of joint rotations within each sample of five rotations was, on average, in the order of 6.5 degrees. In addition, comparing means of limb pose in the pairs 2-5 and 3-6, two situations with kinematics and mechanics apparently similar, it was found that 62.5 percent of the tests were significant. Other variables than kinematics and mechanics might not have been properly controlled. For instance, it was found that variations in drill orientation was as much as 15 degrees (Rancourt, 1995, p. 146). Clearly, this could explain the high variability in limb pose for the drilling task. Pose variations between locations 2-5 and 3-6 could be explained by a systematic drill orientation variation. In summary, evidence of a selection criterion for pose in a drilling task may be difficult to demonstrate because of the variability in drill operation. Results obtained in this study do show however that variations are somewhat small and may not significantly influence several criteria for pose (e. g. limb isotropy, Angeles, 1992), maximizing manipulability (Yoshikawa, 1990), shaping hand impedance (Colgate and Hogan, 1988). Experiments to test the existence of a selection criterion for pose should thus be performed with a task precisely defined in all 6 degrees of freedom of the hand.

In addition, a limited training in drill operation was given to the human subjects before experimental data were collected. Some of the subjects had a significant past experience in drilling while others not. This inter-subject variation was also observed in the data where consistency in limb pose for a given drilling location varied across subjects. Since an optimization criterion may only develop with usage, it would be interesting to conduct the experiment again with skilled drill operators only.

Relative orientation information at each upper limb joint was used in this paper as means to characterize upper limb pose. In fact, a complete description of joint

motion requires both information on rotation and translation. In the case of the wrist and the elbow, one may assume that rotation and translation are related, and hence, characterizing pose by orientation statistics could be sufficient. This is probably not true in the case of the shoulder if one includes the motion of the scapula in the analysis. The addition of translation changes to the analysis that has been presented is worthy of further investigations.

Appendix

Proof of Proposition 1

Elementary calculations show that $\text{tr}\{\Phi(b)\Phi(a)\} = -2a^tb$ and $\Phi(b)^2 = -b^tbI + bb^t$, where a and b are 3×1 vectors. Thus,

$$\bar{X} = M\{I + \Phi(\bar{\varepsilon}) - \sum(\varepsilon_i^t\varepsilon_i I - \varepsilon_i\varepsilon_i^t)/2n\} + o_p(\kappa^{-1}).$$

One has $\hat{M} = M \exp\{\Phi(m)\}$, where m is a 3×1 $O_p(\kappa^{-1/2})$ vector such that $\text{tr}(\hat{M}^t\bar{X})$ is maximum. Now

$$\text{tr}(\hat{M}^t\bar{X}) = 3 - \text{tr}\{\Phi(m)\Phi(\bar{\varepsilon})\} - \sum(\varepsilon_i^t\varepsilon_i)/n - m^tm + o_p(\kappa^{-1}).$$

This is equivalent to

$$\text{tr}(\hat{M}^t\bar{X}) = 3 - \sum(\varepsilon_i - m)^t(\varepsilon_i - m)/n + o_p(\kappa^{-1}).$$

Thus $m = \bar{\varepsilon}$ minimizes the $O_p(\kappa^{-1})$ component of the second term of the above expression. Therefore $\hat{M} = M \exp\{\Phi(\bar{\varepsilon})\} + O_p(\kappa^{-1})$. This proves the expansion for \hat{M} . The expansion for the residual comes from the fact that $\hat{M}^t X_i = I + \Phi(\varepsilon_i - \bar{\varepsilon}) + O_p(\kappa^{-1})$.

Quaternion implementation of the analysis

Let X_1 and X_2 be two rotation matrices with respective quaternion representations q_1 and q_2 . To prove that $\text{tr}(X_1^t X_2) = 4(q_1^t q_2)^2 - 1$, let θ be the angle of the rotation $X_1^t X_2$. From (2.2), $\text{tr}(X_1^t X_2) = 2 \cos(\theta) + 1 = 4 \cos(\theta/2)^2 - 1$ while $q_1^t q_2$ is the first element of the quaternion representation of $(X_1^t X_2)$, thus $q_1^t q_2 = \cos(\theta/2)$.

From (2.5) and (2.2), the residual r_i can be expressed as $\sin(\theta)u$, where u and θ are respectively the axis and the angle of the rotation $\hat{M}^t X_i$. The quaternion for this rotation is $R_i = (\cos(\theta/2), \sin(\theta/2)u^t)^t$. The formula $r_i = 2R_{i1}(R_{i2}, R_{i3}, R_{i4})^t$ comes from applying the standard result that $\sin(\theta) = 2 \sin(\theta/2) \cos(\theta/2)$.

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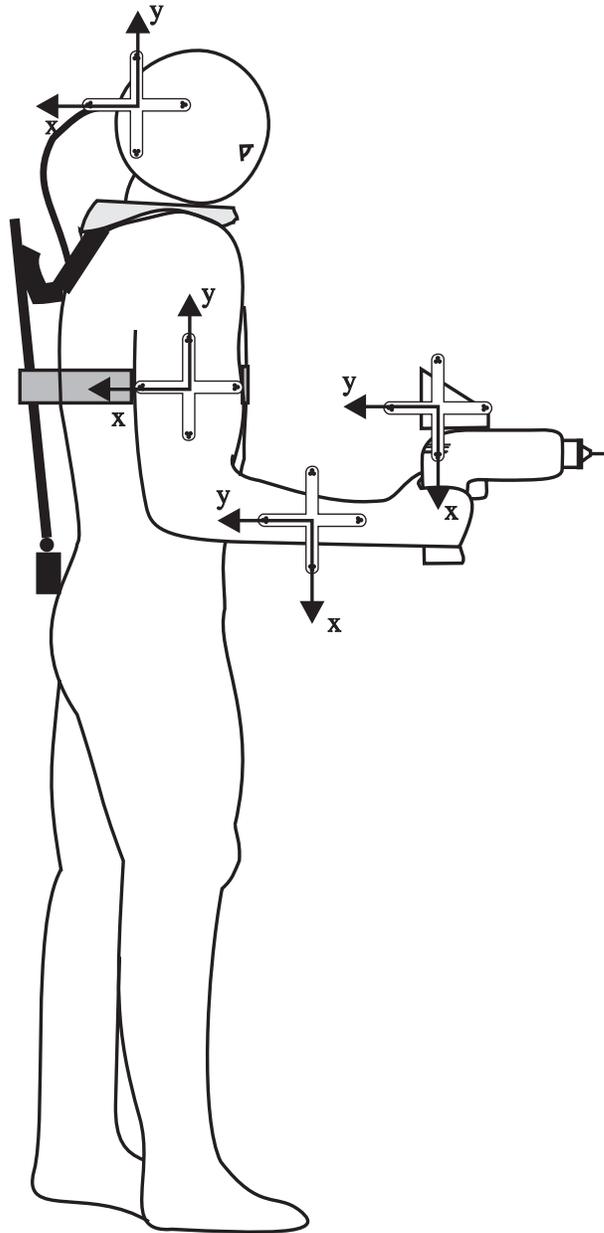


Figure 1: Location and Orientation of Markers in the Drilling Task Investigated. For all markers, the z-axis points forward

Fixed Vertical Panel

Metal Plate

Drill

Marker

Infrared camera

Infrared camera

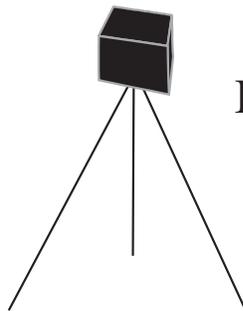
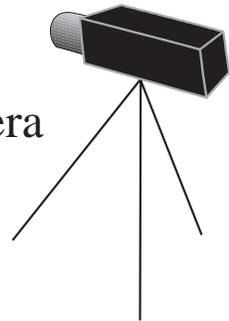
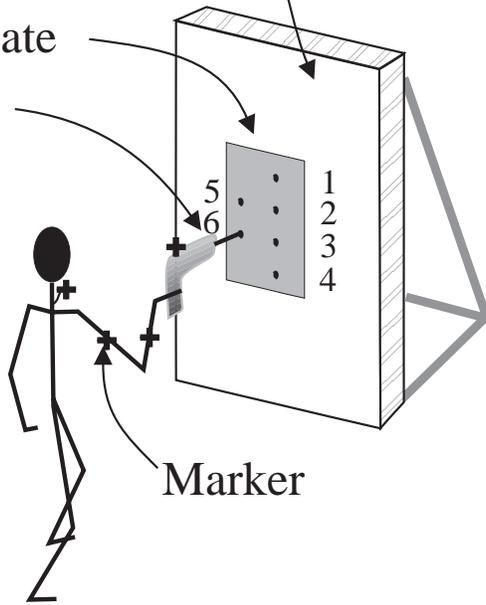


Figure 2: The Six Drilling Positions

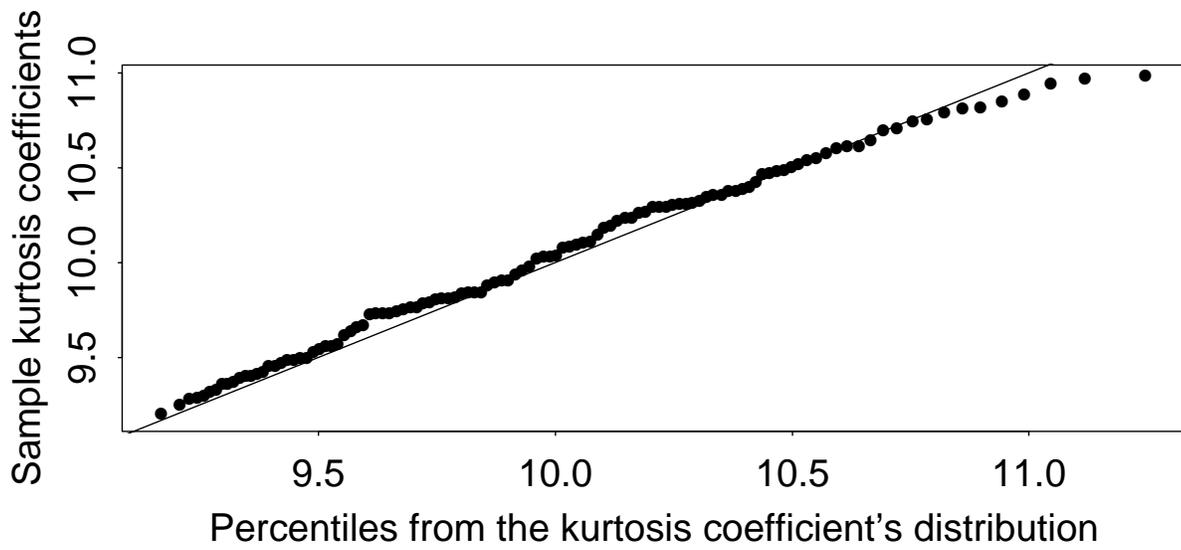
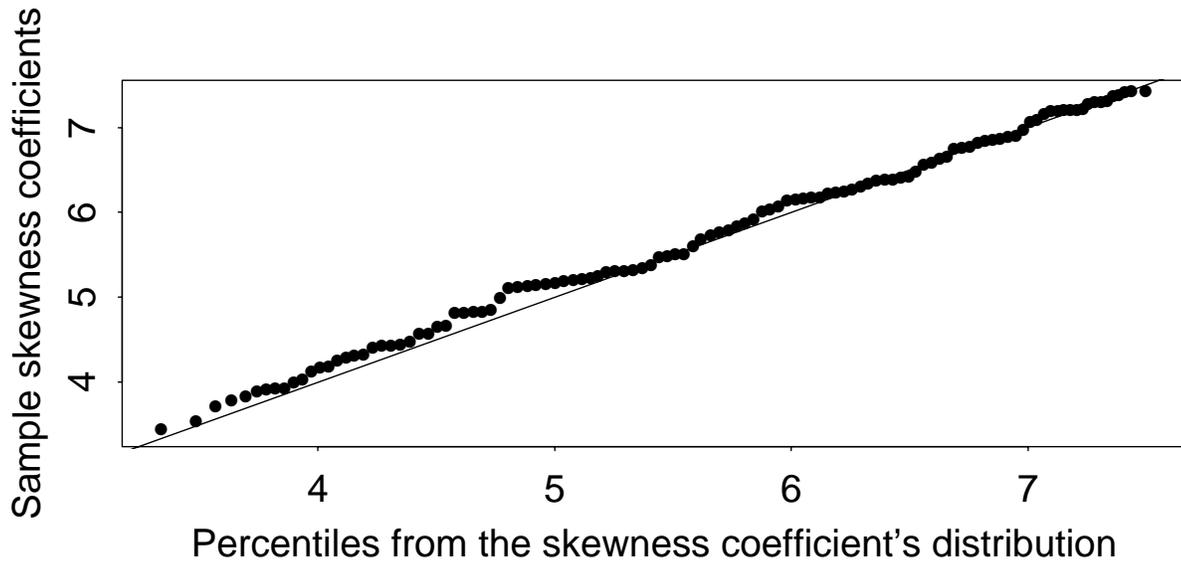


Figure 3: QQ-plots for normality of the 129 samples. Each sample is represented by a point.